

# Comments on a Flavor Symmetry \*

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## 1. Introduction

We know that the masses of the charged fermions rapidly increase as  $(u, d, e) \rightarrow (c, s, \mu) \rightarrow (t, b, \tau)$ . At first, it has been considered that such rapid increasing of the mass spectra cannot be understood from an idea of “symmetry”. The horizontal degree of freedom has been called as “generations”. In contrast to the idea of “generations”, there is an idea of “families” that the horizontal quantum number states have basically the same opportunity. After the democratic mass matrix model [1] was proposed, the idea of “families” became one of the promising viewpoints for “flavors”. Nowadays, a popular idea to understand the observed quark and lepton mass spectra and mixing matrices is to assume a flavor symmetry which puts constraints on the Yukawa coupling constants.

We sometimes take a phenomenological approach “symmetry + its breaking” for the fermion mass matrices. For example, in order to understand the neutrino masses and mixings, we put a flavor symmetry on the neutrino mass matrix on the flavor basis where the charged lepton mass matrix is diagonal. However, we must remember that the left-handed neutrino  $\nu_{Li}$  is a partner of the left-handed charged lepton  $e_{Li}$  of the  $SU(2)_L$  doublet. We usually consider that the  $SU(2)_L$  symmetry is unbroken until a low energy (electroweak) scale  $M_W$ . Therefore, if we consider a flavor symmetry under which the fields  $\nu_L$  and  $e_L$  are transformed as  $\nu_L = U_X \nu'_L$  and  $e_L = U_X e'_L$ , the neutrino and charged lepton mass matrices  $M_\nu$  and  $M_e$  must satisfy the relations

$$U_X^\dagger M_\nu U_X^* = M_\nu, \quad U_X^\dagger M_e M_e^\dagger U_X = M_e M_e^\dagger, \quad (1)$$

for the energy scale  $\mu > M_W$ . Also, the up- and down-quark mass matrices  $M_u$  and  $M_d$  must satisfy the relations

$$U_X^\dagger M_u M_u^\dagger U_X = M_u M_u^\dagger, \quad U_X^\dagger M_d M_d^\dagger U_X = M_d M_d^\dagger. \quad (2)$$

In the present talk, we will point out [2] that if a flavor symmetry (a discrete symmetry, a  $U(1)$  symmetry, and so on) exists, we cannot obtain the observed Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix  $V_q$  and Maki-Nakagawa-Sakata (MNS) lepton mixing matrix  $U_\ell$ , even if we can obtain reasonable mass spectra under the symmetry. Such the serious constraint is derived only from the relations (1) [and also (2)], and we will not assume any explicit flavor symmetry and/or any explicit mass matrix forms. And then, we will discuss the meaning of the severe result.

## 2. What happens if a flavor symmetry exists?

First, we investigate relations in the quark sectors under the conditions (2). The Hermitian matrix  $M_f M_f^\dagger$  ( $f = u, d$ ) is, in general, diagonalized as

$$(U_L^f)^\dagger M_f M_f^\dagger U_L^f = D_f^2 \equiv \text{diag}(m_{f1}^2, m_{f2}^2, m_{f3}^2), \quad (3)$$

and the CKM matrix  $V_q$  is given by  $V_q \equiv (U_L^u)^\dagger U_L^d$ . From Eqs. (2) and (3), we obtain the relation  $(U_X^f)^\dagger D_f^2 U_X^f = D_f^2$ , where  $U_X^f = (U_L^f)^\dagger U_X U_L^f$ . Therefore, the matrix  $U_X^f$  must be a diagonal matrix with

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a form  $U_X^f = P_X^f \equiv \text{diag}(e^{i\delta_1^f}, e^{i\delta_2^f}, e^{i\delta_3^f})$ , unless the masses are not degenerated. Therefore, we obtain  $U_X = U_L^u P_X^u (U_L^u)^\dagger = U_L^d P_X^d (U_L^d)^\dagger$ , which leads to a constraint on the CKM matrix:  $P_X^u = V_q P_X^d (V_q)^\dagger$ , i.e.

$$(e^{i\delta_i^u} - e^{i\delta_j^d})(V_q)_{ij} = 0 \quad (i, j = 1, 2, 3). \quad (4)$$

Only when  $\delta_i^u = \delta_j^d$ , we can obtain  $(V_q)_{ij} \neq 0$ . Since we do not consider a trivial case with  $U_X \propto \mathbf{1}$ , we cannot consider such a case as all elements of  $V_q$  are not zero. We can obtain only the CKM mixing between two families.

For the lepton sectors, the situation is the same. From Eqs. (1) and (3), we again obtain a severe constraint on the MNS mixing matrix  $U_\ell$ :

$$(e^{i\delta_i^e} - e^{i\delta_j^\nu})(U_\ell)_{ij} = 0 \quad (i, j = 1, 2, 3). \quad (5)$$

### 3. How to evade this trouble

One way to evade the present severe conclusions (4) and (5) is to adopt a model with no flavor symmetry. We consider that each generation has a hierarchically different structure, so that the fermion flavors are ones which should be understood from the concept of “generations” rather than from that of “families”. For example, in the Froggatt and Nielsen model [3], the flavor has a “generation” structure, so that the model should be regarded as a model with “no flavor symmetry”, although the model is based on a U(1) symmetry at a high energy scale. The model can evade the present trouble, and it is indeed one of the most promising models which can reasonably understand the generations.

However, we know the fact (the degree of freedom of “rebasings”) that we cannot physically distinguish two mass matrix sets  $(M_u, M_d)$  and  $(M'_u, M'_d)$ , where  $(M'_u, M'_d)$  is obtained from  $(M_u, M_d)$  by a common flavor-basis rotation for the  $SU(2)_L$  doublet fields. (The situation is the same in the lepton sector.) Only when there is a flavor symmetry, the mass matrix forms  $(M_u, M_d)$  in a specific flavor basis have a meaning, because the operator of the flavor rotation does not commute with the flavor symmetry operator  $U_X$ . Therefore, the idea of a flavor symmetry is still attractive to most mass-matrix-model-builders.

In order to evade the troubles (4) and (5) within the framework of the “families” (not “generations”), we have to seek for a flavor symmetry breaking mechanism with the following conditions: (i) The original Lagrangian (including the symmetry breaking mechanism) is exactly invariant under the  $SU(2)_L$ . (ii) The flavor symmetry should be completely broken at a high energy scale  $M_X$ , so that we cannot have any flavor symmetry below  $\mu = M_X$ .

For example, let us consider a two Higgs doublet model, or a  $\bar{5}_L \leftrightarrow \bar{5}'_L$  model. In such a model, the effective Yukawa coupling constants  $Y^f$  below  $\mu = M_X$  are given by a linear combination of two Yukawa coupling constants with different textures  $Y_A^f$  and  $Y_B^f$ ,

$$Y^f = c_A^f Y_A^f + c_B^f Y_B^f, \quad (6)$$

so that  $Y^f (Y^f)^\dagger$  do not satisfy the flavor symmetry conditions (1) [or (2)]. However, we should note that the matrices  $Y_A^f (Y_A^f)^\dagger$  and  $Y_B^f (Y_B^f)^\dagger$  have to satisfy the  $SU(2)_L$  constraints individually. Regrettably, Some of currently proposed models with a phenomenological flavor symmetry breaking seem to be unconcerned about this  $SU(2)_L$  constraints.

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[2] Y. Koide, hep-ph/0406286.

[3] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147** (1979) 277.